$$\int \frac{e^{2x}}{1+e^{2x}}\,dx = \int \frac{1}{u}\cdot\frac{du}{2}$$

$$= \frac{1}{2}\int \frac{1}{u}\,du$$

$$= \frac{1}{2}\ln|u| + C$$

$$= \frac{1}{2}\ln\left(1+e^{2x}\right) + C$$

$u = 1+e^{2x}$

$du = e^{2x}\cdot 2\,dx$

$\frac{du}{2} = e^{2x}\,dx$

Check:

$$y = \frac{1}{2}\ln\left(1+e^{2x}\right)+C$$

$$\frac{dy}{dx} = \frac{1}{2}\cdot\frac{1}{1+e^{2x}}\cdot e^{2x}\cdot 2$$

$$\frac{dy}{dx} = \frac{e^{2x}}{1+e^{2x}}$$

$$\int x^2 e^{x^3} dx = \int e^u \cdot \frac{du}{3}$$

$u = x^3$

$du = 3x^2 dx$

$\frac{du}{3} = x^2 dx$

$= \frac{1}{3}\int e^u du$

$= \frac{1}{3} e^u + C$

$y = \frac{1}{3} e^{x^3} + C$

$$\int (e^x - e^{-x})\,dx = e^x + \int -e^{-x}\,dx$$

$u = -x$

$du = -1\,dx$

$-du = dx$

$du = -dx$

$$= e^x + \int e^u\,du$$

$$= e^x + e^u + C$$

$$y = e^x + e^{-x} + C$$

Check:

$$\frac{dy}{dx} = e^x + e^{-x} \cdot (-1)$$

$$= e^x - e^{-x}$$

$$\int e^{\sin \pi x} \cos \pi x \, dx = \int e^{u} \frac{du}{\pi}$$

$u = \sin \pi x$

$du = \cos \pi x \cdot \pi \, dx$

$$\frac{du}{\pi} = \cos \pi x \, dx$$

$$= \frac{1}{\pi} \int e^{u} \, du$$

$$y = \frac{1}{\pi} e^{u} + C$$

$$y = \frac{1}{\pi} e^{\sin \pi x} + C$$

$$\int \frac{e^x + e^{-x}}{e^x - e^{-x}}\,dx = \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$u = e^x - e^{-x}$

$du = e^x - (e^{-x}) \cdot (-1)\,dx$

$du = (e^x + e^{-x})\,dx$

$y = \ln|e^x - e^{-x}| + C$

12. $\int \frac{(\ln x)^2}{x}\,dx = \int u^2\,du$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$= \frac{u^3}{3} + C$

$y = \frac{(\ln x)^3}{3} + C$

$$\int \frac{\sec x \tan x}{\sec x - 1}\, dx = \int \frac{1}{u}\, du$$

$$u = \sec x - 1$$

$$du = \sec x \tan x\, dx$$

$$y = \ln|u| + C$$

$$y = \ln|\sec x - 1| + C$$

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